American University of Beirut

Exam I
MECH 320, Spring 08

Saturday, March 29, 2008
Duration: 90 minutes

Miscellaneous Formulas:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_y = \frac{\sigma_{x'} + \sigma_{y'}}{2} - \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta - \tau_{x'y'}\sin 2\theta$$

$$\sigma_x = \frac{\sigma_{x'} + \sigma_{y'}}{2} + \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta + \tau_{x'y'}\sin 2\theta$$

$$\tau_{xy} = -\frac{\sigma_{x'} - \sigma_{y'}}{2}\sin 2\theta + \tau_{x'y'}\cos 2\theta$$

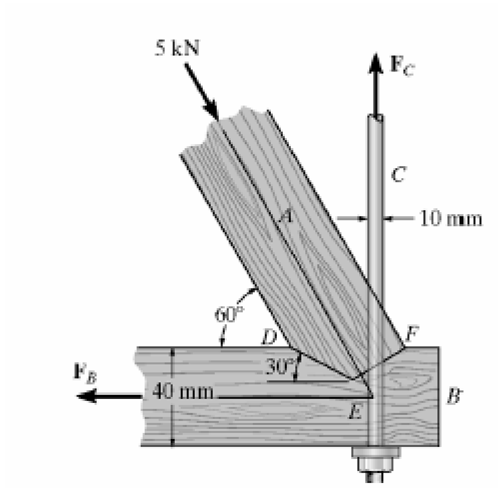
$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\tau_{max_{in\text{-}plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

Problem 1 –Equilibrium Equations (20 points)

Member A of the body shown in the figure is subjected to a compressive force of 5 kN. Determine the average normal stress acting on the member C which has a diameter of 10 mm and in member B which has a thickness of 30 mm.



Equations of Equilibrium :

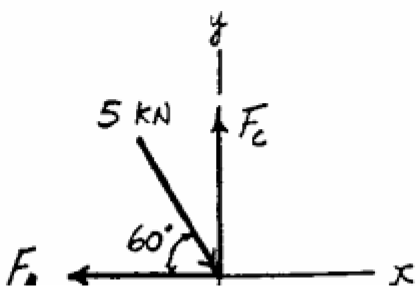
$$\xrightarrow{+} \Sigma F_x = 0; \quad 5\cos 60^\circ - F_B = 0 \quad F_B = 2.50 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad F_c - 5\sin 60^\circ = 0 \quad F_c = 4.330 \text{ kN}$$

Average Normal Stress :

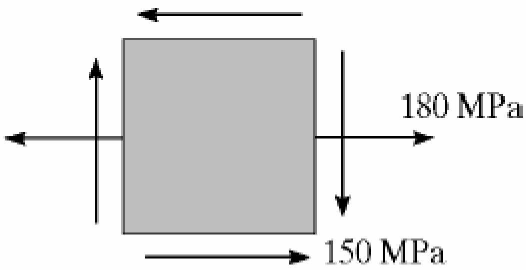
$$\sigma_B = \frac{F_B}{A_B} = \frac{2.50(10^3)}{(0.04)(0.03)} = 2.08 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_C = \frac{F_C}{A_C} = \frac{4.330(10^3)}{\frac{\pi}{4}(0.01^2)} = 55.1 \text{ MPa} \qquad \textbf{Ans}$$



Problem 2- Principal stresses, shear stress and orientations (20 points)

The state of stress at a point is shown on the element. Determine (a) the principal stresses (b) the maximum in plane shear stress and average normal stress at the point. Determine the values of θ_p and θs.

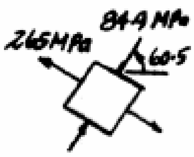


$\sigma_x = 180$ MPa $\qquad \sigma_y = 0 \qquad \tau_{xy} = -150$ MPa

a) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{180+0}{2} \pm \sqrt{\left(\frac{180-0}{2}\right)^2 + (-150)^2}$$

$\sigma_1 = 265$ MPa **Ans** $\qquad \sigma_2 = -84.9$ MPa **Ans**



Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-150}{(180-0)/2} = -1.6667$$

$\theta_p = 60.482°$ and $-29.518°$

Use Eq. 9 - 1 to determine the pricipal plane of σ_1 and σ_2:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \qquad \text{where } \theta = 60.482°$$

$$= \frac{180+0}{2} + \frac{180-0}{2}\cos 2(60.482°) + (-150)\sin 2(60.482°) = -84.9 \text{ MPa}$$

Therefore $\theta_{p1} = 60.5°$ **Ans** and $\theta_{p2} = -29.5°$ **Ans**

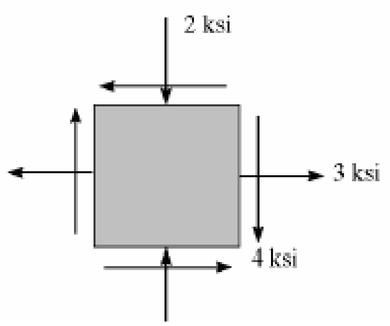
b)

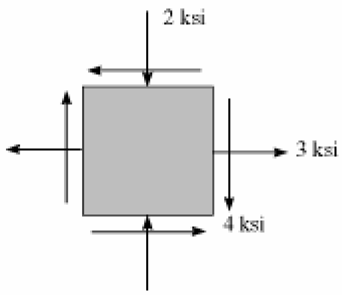
$$\tau_{max_{in\text{-}plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{180-0}{2}\right)^2 + (-150)^2} = 175 \text{ MPa} \quad \textbf{Ans}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{180+0}{2} = 90.0 \text{ MPa} \quad \textbf{Ans}$$

Problem 3- (Use Mohr's circle for this problem)-20 pts

Determine the equivalent state of stress if an element is oriented 20° clockwise from the element shown. Show the result on the element.





Construction of the Circle: In accordance with the sign convention, $\sigma_x = 3$ ksi, $\sigma_y = -2$ ksi and $\tau_{xy} = -4$ ksi. Hence,

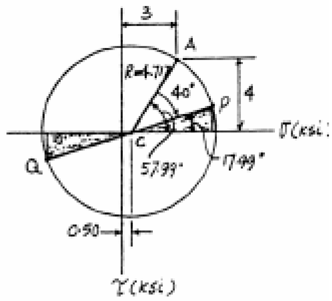
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{3 + (-2)}{2} = 0.500 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(3,\ -4) \qquad C(0.500,\ 0)$$

The radius of the circle is

$$R = \sqrt{(3 - 0.500)^2 + 4^2} = 4.717 \text{ ksi}$$

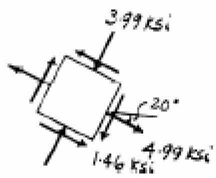


Stress on The Rotated Element: The normal and shear stress components ($\sigma_{x'}$ and $\tau_{x'y'}$) are represented by the coordinate of point P on the circle. $\sigma_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

$\sigma_{x'} = 0.500 + 4.717\cos 17.99° = 4.99$ ksi **Ans**

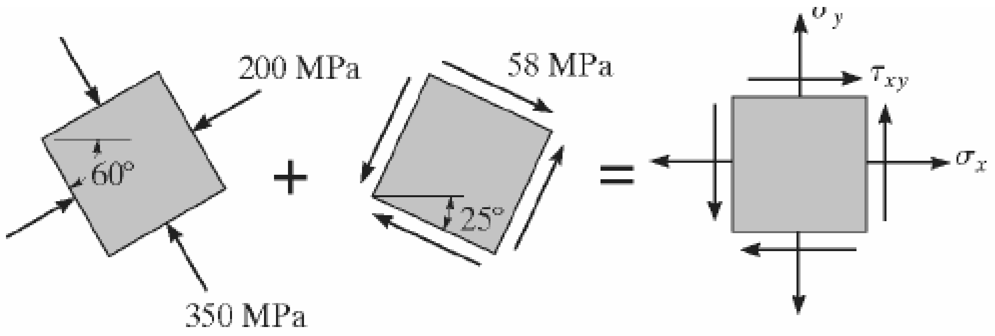
$\tau_{x'y'} = -4.717\sin 17.99° = -1.46$ ksi **Ans**

$\sigma_{y'} = 0.500 - 4.717\cos 17.99° = -3.99$ ksi **Ans**

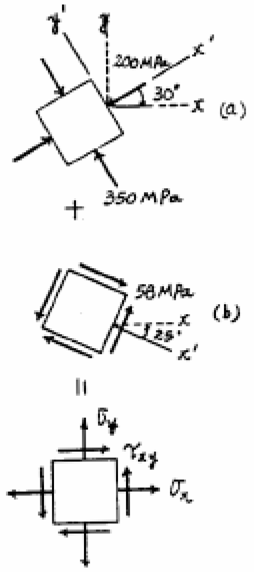


Problem 4- Stress-20 pts

A point on a thin plate is subjected to the 2 successive states of stress shown. Determine the resultant state of stress represented on the element oriented as shown in the right.



(*a*) with $\theta = -30°$, $\sigma_{x'} = -200$ MPa, $\sigma_{y'} = -350$ MPa and $\tau_{x'y'} = 0$,



$$(\sigma_x)_a = \frac{\sigma_{x'}+\sigma_{y'}}{2} + \frac{\sigma_{x'}-\sigma_{y'}}{2}\cos 2\theta + \tau_{x'y'}\sin 2\theta$$
$$= \frac{-200+(-350)}{2} + \frac{-200-(-350)}{2}\cos(-60°) + 0$$
$$= -237.5 \text{ MPa}$$

$$(\sigma_y)_a = \frac{\sigma_{x'}+\sigma_{y'}}{2} - \frac{\sigma_{x'}-\sigma_{y'}}{2}\cos 2\theta - \tau_{x'y'}\sin 2\theta$$
$$= \frac{-200+(-350)}{2} - \frac{-200-(-350)}{2}\cos(-60°) - 0$$
$$= -312.5 \text{ MPa}$$

$$(\tau_{xy})_a = -\frac{\sigma_{x'}-\sigma_{y'}}{2}\sin 2\theta + \tau_{x'y'}\cos 2\theta$$
$$= -\frac{-200-(-350)}{2}\sin(-60°) + 0$$
$$= 64.95 \text{ MPa}$$

For element (*b*), $\theta = 25°$, $\sigma_{x'} = \sigma_{y'} = 0$ and $\tau_{x'y'} = 58$ MPa,

$$(\sigma_x)_b = \frac{\sigma_{x'}+\sigma_{y'}}{2} + \frac{\sigma_{x'}-\sigma_{y'}}{2}\cos 2\theta + \tau_{x'y'}\sin 2\theta$$
$$= 0 + 0 + 58\sin 50°$$
$$= 44.43 \text{ MPa}$$

$$(\sigma_y)_b = \frac{\sigma_{x'}+\sigma_{y'}}{2} - \frac{\sigma_{x'}-\sigma_{y'}}{2}\cos 2\theta - \tau_{x'y'}\sin 2\theta$$
$$= 0 - 0 - 58\sin 50°$$
$$= -44.43 \text{ MPa}$$

$$(\tau_{xy})_b = -\frac{\sigma_{x'}-\sigma_{y'}}{2}\sin 2\theta + \tau_{x'y'}\cos 2\theta$$
$$= -0 + 58\cos 50°$$
$$= 37.28 \text{ MPa}$$

Combining the stress components of two elements yields

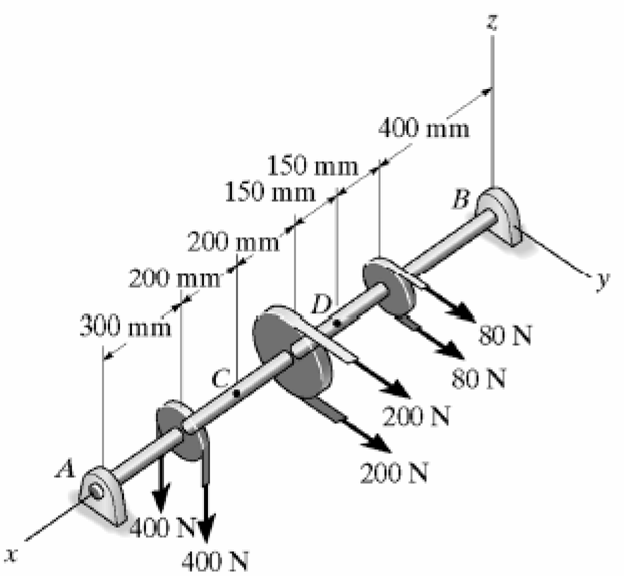
$\sigma_x = (\sigma_x)_a + (\sigma_x)_b = -237.5 + 44.43 = -193$ MPa **Ans**

$\sigma_y = (\sigma_y)_a + (\sigma_y)_b = -312.5 - 44.43 = -357$ MPa **Ans**

$\tau_{xy} = (\tau_{xy})_a + (\tau_{xy})_b = 64.95 + 37.28 = 102$ MPa **Ans**

Problem 5-Equilibrium- 20 pts

The shaft is supported at its ends by two bearings A and B and is subjected to the forces applied to the pulleys fixed to the shaft. Determine the resultant internal loadings acting on the cross section through point C. The 400-N forces act in the $-z$ direction and the 200-N and 80-N forces act in the $+y$ direction. The journal bearings at A and B exert only y and z components of force on the shaft.



Support Reactions :

$\Sigma M_z = 0;\quad 160(0.4) + 400(0.7) - A_y(1.4) = 0$

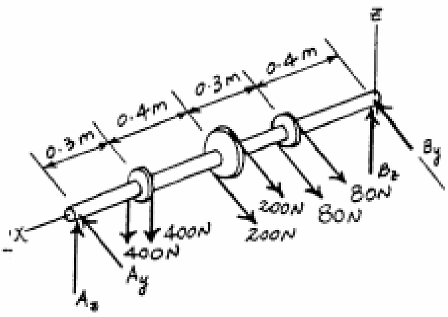
$A_y = 245.71\text{ N}$

$\Sigma F_y = 0;\quad -245.71 - B_y + 400 + 160 = 0$

$B_y = 314.29\text{ N}$

$\Sigma M_y = 0;\quad 800(1.1) - A_z(1.4) = 0 \quad A_z = 628.57\text{ N}$

$\Sigma F_z = 0;\quad B_z + 628.57 - 800 = 0 \quad B_z = 171.43\text{ N}$



Equations of Equilibrium : For point C

$\Sigma F_x = 0;\quad (N_C)_x = 0$ **Ans**

$\Sigma F_y = 0;\quad -245.71 + (V_C)_y = 0$

$(V_C)_y = -246\text{ N}$ **Ans**

$\Sigma F_z = 0;\quad 628.57 - 800 + (V_C)_z = 0$

$(V_C)_z = -171\text{ N}$ **Ans**

$\Sigma M_x = 0;\quad (T_C)_x = 0$ **Ans**

$\Sigma M_y = 0;\quad (M_C)_y - 628.57(0.5) + 800(0.2) = 0$

$(M_C)_y = -154\text{ N}\cdot\text{m}$ **Ans**

$\Sigma M_z = 0;\quad (M_C)_z - 245.71(0.5) = 0$

$(M_C)_z = -123\text{ N}\cdot\text{m}$ **Ans**